Appendix

Inductor

Voltage/current relationship: $v = L\frac{di}{dt}$

Stored energy: $w = \frac{1}{2}Li^2$

Capacitor

Voltage/current relationship: $i = C\frac{dv}{dt}$

Stored energy: $w = \frac{1}{2}Cv^2$

RL Circuits

$$i(t) = I_f + (I_o - I_f)e^{-\frac{t}{\tau}}$$
$$I_f = i(\infty)$$
$$I_o = i(0)$$
$$\tau = \frac{L}{R}$$

RC Circuits

$$v(t) = V_f + (V_o - V_f)e^{-\frac{t}{\tau}}$$
$$V_f = v(\infty)$$
$$V_o = v(0)$$
$$\tau = RC$$

Parallel RLC Circuit

Differential equation: $\frac{d^2x}{dt^2} + \frac{1}{RC}\frac{dx}{dt} + \frac{x}{LC} = 0$, where x=i or x=v

Characteristic equation: $s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$

Solution: $x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + x_f$,

where s_1 and s_2 are the roots of the characteristic equation

Solution: $x(t) = e^{\mathrm{Re}\{s_1\}t}\left[A_1 \cos(\mathrm{Im}\{s_1\}t) + A_2 \sin(\mathrm{Im}\{s_1\}t)\right] + x_f$,

where s_1 and s_2 are the roots of the characteristic equation and are complex conjugates.

Series RLC Circuit

Differential equation: $\frac{d^2x}{dt^2}+\frac{R}{L}\frac{dx}{dt}+\frac{x}{LC}=0$, where $x=i$ or $x=v$

Characteristic equation: $s^2+\frac{R}{L}s+\frac{1}{LC}=0$

Solution: $x(t)=A_1\,e^{s_1 t}+A_2\,e^{s_2 t}+x_f$, where s_1 and s_2 are the roots of the characteristic equation

Solution: $x(t)=e^{\mathrm{Re}\{s_1\}t}\left[A_1\cos(\mathrm{Im}\{s_1\}t)+A_2\sin(\mathrm{Im}\{s_1\}t)\right]+x_f$,

where s_1 and s_2 are the roots of the characteristic equation and are complex conjugates.

Euler's Identity:

$$e^{j\theta}=\cos(\theta)+j\sin(\theta)$$

$$\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\sin(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Sinusoidal Power: $p(t)=P(1+\cos 2\omega t)-Q\sin 2\omega t$

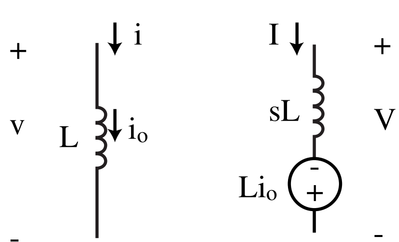
Average Power: $P=\frac{V_m I_m}{2}\cos(\theta_v-\theta_i)$

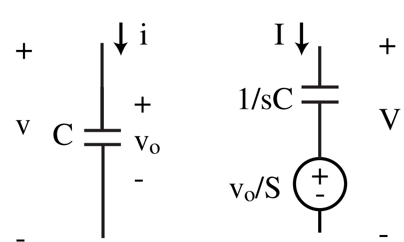
Reactive Power: $Q=\frac{V_m I_m}{2}\sin(\theta_v-\theta_i)$

Apparent Power:

$$\begin{aligned} S &= P+jQ \\ &= \frac{1}{2}\tilde{V}\,\tilde{I}^* \\ &= \frac{1}{2}\frac{\left|\tilde{V}\right|^2}{Z^*} \\ & \quad \frac{1}{2}\left|\tilde{I}\right|^2 Z \end{aligned}$$

f(t)		F(s)
$\delta(t)$	impulse	1
$u(t)$	step	$\frac{1}{s}$
t	ramp	$\frac{1}{s^2}$
e^{-at}	Exponential	$\frac{1}{s+a}$
$\sin(\omega t)$	Sine	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	Cosine	$\frac{s}{s^2+\omega^2}$
$t\,e^{-at}$	Damped ramp	$\frac{1}{(s+a)^2}$
$e^{-at}\sin(\omega t)$	Damped sine	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	Damped cosine	$\frac{s+a}{(s+a)^2+\omega^2}$
$K\,f(t)$	Scale factor	$K\,F(s)$
$\frac{df(t)}{d\,t}$	Derivative	$s\,F(S)-f(0^-)$
$\frac{d^2f(t)}{d\,t^2}$	Second Derivative	$s^2\,F(S)-s\,f(0^-)-\frac{df(0^-)}{dt}$
$\int_0^t f(x)dx$	Integral	$\frac{F(s)}{s}$
$e^{-at}f(t)$		$F(s+a)$
$t\,f(t)$		$-\frac{dF(s)}{ds}$
$f(t-a)\,u(t-a)$	Time shift	$e^{-as}F(s)$





$$v = L\frac{di}{dt} \qquad V = sLI - Li_o$$

$$v = C\frac{dv}{dt} \qquad V = \frac{I}{sC} + \frac{v_o}{s}$$