

Appendix

Inductor

$$\text{Voltage/current relationship: } v = L \frac{di}{dt}$$

$$\text{Stored energy: } w = \frac{1}{2} L i^2$$

Capacitor

$$\text{Voltage/current relationship: } i = C \frac{dv}{dt}$$

$$\text{Stored energy: } w = \frac{1}{2} C v^2$$

RL Circuits

$$i(t) = I_f + (I_o - I_f) e^{-\frac{t}{\tau}}$$

$$I_f = i(\infty)$$

$$I_o = i(0)$$

$$\tau = \frac{L}{R}$$

RC Circuits

$$v(t) = V_f + (V_o - V_f) e^{-\frac{t}{\tau}}$$

$$V_f = v(\infty)$$

$$V_o = v(0)$$

$$\tau = RC$$

Parallel RLC Circuit

$$\text{Differential equation: } \frac{d^2 x}{dt^2} + \frac{1}{RC} \frac{dx}{dt} + \frac{x}{LC} = 0, \text{ where } x=i \text{ or } x=v$$

$$\text{Characteristic equation: } s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\text{Solution: } x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + x_f,$$

where s_1 and s_2 are the roots of the characteristic equation

$$\text{Solution: } x(t) = e^{\text{Re}\{s_1\}t} [A_1 \cos(\text{Im}\{s_1\}t) + A_2 \sin(\text{Im}\{s_1\}t)] + x_f,$$

where s_1 and s_2 are the roots of the characteristic equation and are complex conjugates.

Series RLC Circuit

Differential equation: $\frac{d^2x}{dt^2} + \frac{R}{L} \frac{dx}{dt} + \frac{x}{LC} = 0$, where $x=i$ or $x=v$

Characteristic equation: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

Solution: $x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + x_f$, where s_1 and s_2 are the roots of the characteristic equation

Solution: $x(t) = e^{\text{Re}\{s_1\}t} [A_1 \cos(\text{Im}\{s_1\}t) + A_2 \sin(\text{Im}\{s_1\}t)] + x_f$,
where s_1 and s_2 are the roots of the characteristic equation and are complex conjugates.

Euler's Identity: $e^{j\theta} = \cos(\theta) + j \sin(\theta)$

Sinusoidal Power: $p(t) = P(1 + \cos 2\omega t) - Q \sin 2\omega t$

Average Power: $P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

Reactive Power: $Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$

Apparent Power:

$$\begin{aligned} S &= P + jQ \\ &= \frac{1}{2} \tilde{V} \tilde{I}^* \\ &= \frac{1}{2} \frac{|\tilde{V}|^2}{Z^*} \\ &= \frac{1}{2} |\tilde{I}|^2 Z \end{aligned}$$